

# A leading-twist beam-spin asymmetry in double-hadron inclusive lepto-production

Mauro Anselmino,<sup>1</sup> Vincenzo Barone,<sup>2</sup> and Aram Kotzinian<sup>1,3</sup>

<sup>1</sup>*Dipartimento di Fisica Teorica, Università di Torino;  
INFN, Sezione di Torino, 10125 Torino, Italy*

<sup>2</sup>*Di.S.T.A., Università del Piemonte Orientale “A. Avogadro”;  
INFN, Gruppo Collegato di Alessandria, 15121 Alessandria, Italy*

<sup>3</sup>*Yerevan Physics Institute, 375036 Yerevan, Armenia*

We show that a new beam-spin asymmetry appears in deep inelastic inclusive lepto-production at low transverse momenta when a hadron in the target fragmentation region is observed in association with another hadron in the current fragmentation region. The beam leptons are longitudinally polarized while the target nucleons are unpolarized. This asymmetry is a leading-twist effect generated by the correlation between the transverse momentum of quarks and the transverse momentum of the hadron emitted by the target. Experimental signatures of this effect are discussed.

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Polarization phenomena in inclusive lepto-production have recently attracted a huge amount of theoretical and experimental interest; new theoretical concepts have been introduced to interpret the data and several dedicated experiments are either running, being built or designed [1, 2].

According to the usual QCD partonic interpretation the high energy leptons interact with the nucleon ( $N$ ) constituents by exchanging a virtual photon ( $\gamma^*$ ); the final lepton is detected, while the target and the struck quark fragment into final hadrons which may or may not be observed. A rich amount of information can be inferred from the azimuthal distribution, around the  $\gamma^*$ -direction, of the final particles and, eventually, of the nucleon transverse spin.

In particular it has been discovered that correlations between the spin and the transverse momentum of quarks and/or hadrons give rise to various single-spin and azimuthal asymmetries that would be vanishing or negligible if generated by perturbative effects only. Thus, this partonic interpretation, which holds at leading order in  $1/Q$  (leading-twist), yields non perturbative information on the 3-dimensional spin and momentum distribution of quarks inside the protons and neutrons.

One of most interesting observable is the beam-spin asymmetry in semi-inclusive deeply inelastic scattering (SIDIS) with longitudinally polarised leptons and an unpolarized target,  $l^\uparrow N \rightarrow l' h X$ . When the hadron is produced in the current fragmentation region (that is generated by the fragmentation of a struck quark), this asymmetry, usually denoted by  $A_{LU}$ , is characterized by a  $\sin \phi$  modulation, where  $\phi$  is the azimuthal angle of the hadron measured with respect to the lepton scattering plane.

Perturbatively,  $A_{LU}$  is a  $\alpha_s^2$  effect [3] and its magnitude has been estimated to be of order of one per mille [4]. On the other hand, experimental studies [5–7] have shown that  $A_{LU}$  is much larger, of the order of few percents.

This can be explained by taking into account the intrinsic transverse motion of quarks. In fact, transverse momentum dependent distribution and fragmentation functions, which reflect spin-orbit correlations inside hadrons, contribute to  $A_{LU}$  at twist-3, i.e. at  $\mathcal{O}(1/Q)$  level [8, 9]. Predictions of these effects are difficult, as they involve many unknown quantities, but various simple models are able to explain the observed size of  $A_{LU}$  [10, 11]. The peculiarity of  $A_{LU}$  is that it is the only observable that is expected to vanish in the parton model, being entirely due to quark-gluon interactions.

The purpose of this Letter is to point out that, if a second hadron is detected in the target fragmentation region, a new beam-spin asymmetry arises, which is a *leading-twist* observable. This asymmetry is related to a transverse momentum dependent fracture function [12, 13] describing longitudinally polarized quarks inside an unpolarized nucleon. The fracture functions give the conditional probabilities to find a quark inside a nucleon fragmenting into a final hadron. Upon integration and sum over the final hadron degrees of freedom they reproduce the usual Transverse Momentum Dependent distribution functions (TMDs). The asymmetry turns out to have a typical azimuthal modulation as a function of  $\Delta\phi$ , where  $\Delta\phi = \phi_1 - \phi_2$  is the difference between the azimuthal angles of the hadrons in the current and target fragmentation regions.

The process we are interested in is two-particle inclusive lepto-production with one hadron ( $h_1$ ) in the current fragmentation region (CFR) and one hadron ( $h_2$ ) in the target fragmentation region (TFR),

$$l^\uparrow(\ell) + p(P) \rightarrow l(\ell') + h_1(P_1) + h_2(P_2) + X. \quad (1)$$

We suppose that the incoming lepton is longitudinally polarized, while the nucleon target and both final hadrons are unpolarized.

Single-particle SIDIS is usually described in terms of

the three variables

$$x_B = \frac{Q^2}{2P \cdot q} \quad y = \frac{P \cdot q}{P \cdot \ell} \quad z_1 = \frac{P \cdot P_1}{P \cdot q} \quad (2)$$

When a second hadron,  $h_2$ , is produced, one needs further variables related to  $P_2$ . It is convenient to use a light-cone parametrization of vectors. Given a generic vector  $A^\mu = (A^0, A^1, A^2, A^3)$ , their light-cone components are defined as  $A^\pm \equiv (A^0 \pm A^3)/\sqrt{2}$  and we write  $A^\mu = [A^+, A^-, \mathbf{A}_\perp]$ . We also introduce two null vectors,  $n_+^\mu = [1, 0, \mathbf{0}_\perp]$  and  $n_-^\mu = [0, 1, \mathbf{0}_\perp]$ , with  $n_+ \cdot n_- = 1$ , so that a vector can be parametrized as  $A^\mu = A^+ n_+^\mu + A^- n_-^\mu + A_\perp^\mu$ .

We work in a frame where the target nucleon and the virtual photon are collinear (we call it a “ $\gamma^* N$  collinear frame”). The nucleon is supposed to move in the negative  $z$  direction. The unit vector  $\hat{\mathbf{q}} \equiv \mathbf{q}/|\mathbf{q}|$  identifies the positive  $z$  direction.

In terms of the null vectors  $n_+^\mu$  and  $n_-^\mu$  the four-momenta at hand are (we neglect  $\mathcal{O}(1/Q^2)$  terms):

$$P^\mu = P^- n_-^\mu, \quad (3)$$

$$q^\mu = \frac{Q^2}{2x_B P^-} n_+^\mu - x_B P^- n_-^\mu, \quad (4)$$

$$P_1^\mu = \frac{z_1 Q^2}{2x_B P^-} n_+^\mu + P_{1\perp}^\mu, \quad (5)$$

$$P_2^\mu = \zeta_2 P^- n_-^\mu + P_{2\perp}^\mu. \quad (6)$$

In terms of the variables  $(z_1, \mathbf{P}_{1\perp})$  and  $(\zeta_2, \mathbf{P}_{2\perp})$ , the cross section takes the form [13]

$$\begin{aligned} & \frac{d\sigma}{dx_B dy dz_1 d\zeta_2 d^2\mathbf{P}_{1\perp} d^2\mathbf{P}_{2\perp}} \\ &= \frac{\alpha_{\text{em}}^2}{8(2\pi)^2 Q^4} \frac{y}{z_1 \zeta_2} L_{\mu\nu} W^{\mu\nu}. \end{aligned} \quad (7)$$

$L^{\mu\nu}$  is the usual DIS leptonic tensor, explicitly given in [8] (it can also be found in [13]). The hadronic tensor  $W^{\mu\nu}$  is represented in the parton model, or, equivalently, at lowest order in QCD, by the handbag diagram of Fig. 1. It incorporates the transverse momentum dependent fracture functions introduced in [12], which represent the conditional probability to find a quark with longitudinal momentum fraction  $x_B$  and transverse momentum  $\mathbf{k}_\perp$  inside a nucleon that fragments into a hadron carrying a fraction  $\zeta_2$  of the nucleon longitudinal momentum and a transverse momentum  $\mathbf{P}_{2\perp}$ . The hadronic tensor also incorporates, in this case of double production, the fragmentation function describing the production of the final hadron in the CFR. In particular, for an unpolarized target,  $W^{\mu\nu}$  explicitly reads at leading twist [13]

$$\begin{aligned} W^{\mu\nu} &= 4z_1 \zeta_2 (2\pi)^3 \sum_a e_a^2 \\ &\times \int d^2\mathbf{k}_\perp \int d^2\mathbf{k}'_\perp \delta^2(\mathbf{k}_\perp - \mathbf{k}'_\perp - \mathbf{P}_{1\perp}/z_1) \end{aligned}$$

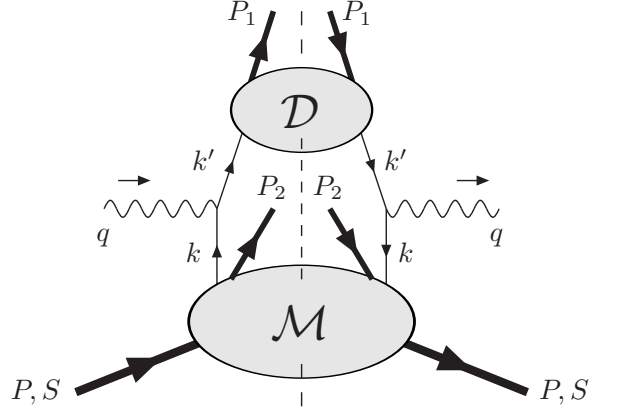


FIG. 1: The handbag diagram for double-hadron lepton production. The lower blob contains the fracture functions and the upper one the fragmentation functions.

$$\begin{aligned} & \times \left\{ -g_\perp^{\mu\nu} \hat{u}_1 D_1 \right. \\ & + \frac{P_{2\perp}^{\{\mu} k_\perp^{\nu\}} - g_\perp^{\mu\nu} P_{2\perp} \cdot k_\perp}{m_1 m_2} \hat{t}_1^h H_1^\perp \\ & + \frac{k_\perp^{\{\mu} k'_\perp{}^{\nu\}} - g_\perp^{\mu\nu} k_\perp \cdot k'_\perp}{m_1 m_N} \hat{t}_1^\perp H_1^\perp \\ & \left. + i \epsilon_\perp^{\mu\nu} \frac{\mathbf{k}_\perp \times \mathbf{P}_{2\perp}}{m_N m_2} \hat{t}_1^{\perp h} D_1 \right\}. \end{aligned} \quad (8)$$

In each term on the r.h.s, a fracture function ( $\hat{u}_1$ ,  $\hat{t}_1^h$ ,  $\hat{t}_1^\perp$ ,  $\hat{t}_1^{\perp h}$ ), which depends on  $x_B$ ,  $\zeta_2$ ,  $\mathbf{k}_\perp^2$ ,  $\mathbf{P}_{2\perp}^2$ ,  $\mathbf{k}_\perp \cdot \mathbf{P}_{2\perp}$ , multiplies a fragmentation function ( $D_1$ ,  $H_1^\perp$ ), which depends on  $z_1$ ,  $\mathbf{k}'^2$ .

A few words about the nomenclature of fracture functions can be helpful. We denote with  $\hat{u}$ ,  $\hat{t}$  and  $\hat{t}^\perp$  the unintegrated fracture functions of unpolarized, longitudinally polarized and transversely polarized quarks, respectively, with the index 1 signaling their leading-twist character and the subscripts  $L$  and  $T$  labeling the polarization of the target (no subscript = unpolarized,  $L$  = longitudinally polarized,  $T$  = transversely polarized). The superscripts  $h$  and  $\perp$  signal the presence of  $P_{2\perp}^i$  and  $k_\perp^i$  accompanying factors, respectively.

As for fragmentation functions,  $D_1$  is the transverse-momentum dependent unpolarized fragmentation function and  $H_1^\perp$  is the Collins function, describing the fragmentation of transversely polarized quarks into a spinless hadron [14].

The first three components of the hadronic tensor (8) give the unpolarized cross section,  $d\sigma_{UU}$ . The fourth one gives the helicity dependent part of the cross section

for a longitudinally polarized lepton beam,  $d\sigma_{LU}$ , and generates a beam-spin asymmetry. Note that there are two terms involving the fracture functions of transversely polarized quarks inside the unpolarized nucleon,  $\hat{t}_1^\perp$  and  $\hat{t}_1^h$ , which are similar to the Boer–Mulders function  $h_1^\perp$  probed in the current fragmentation region [15].

Contracting the hadronic tensor (8) with the leptonic tensor we get the differential cross section (for details of the calculations, see [13, 16]):

$$\begin{aligned} & \frac{d\sigma^{l(\lambda_l) N \rightarrow l h_1 h_2 X}}{dx_B dy dz_1 d\zeta_2 d\mathbf{P}_{1\perp}^2 d\mathbf{P}_{2\perp}^2 d\phi_1 d\phi_2} \\ &= \frac{\pi\alpha_{\text{em}}^2}{x_B y Q^2} \left\{ \left(1 - y + \frac{y^2}{2}\right) \mathcal{F}_{UU} \right. \\ &+ (1 - y) \mathcal{F}_{UU}^{\cos(\phi_1 + \phi_2)} \cos(\phi_1 + \phi_2) \\ &+ (1 - y) \mathcal{F}_{UU}^{\cos(2\phi_1)} \cos(2\phi_1) \\ &+ (1 - y) \mathcal{F}_{UU}^{\cos(2\phi_2)} \cos(2\phi_2) \\ &\left. - \lambda_l y \left(1 - \frac{y}{2}\right) \mathcal{F}_{LU}^{\sin(\phi_1 - \phi_2)} \sin \Delta\phi \right\} \\ &\equiv \sigma_{UU} + \lambda_l \sigma_{LU}, \end{aligned} \quad (9)$$

where  $\lambda_l$  is the lepton helicity and the structure functions  $\mathcal{F}(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2, \mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp})$  are given, at leading twist, by:

$$\mathcal{F}_{UU} = \mathcal{C}[\hat{u}_1 D_1], \quad (10)$$

$$\mathcal{F}_{UU}^{\cos(\phi_1 + \phi_2)} = \frac{|\mathbf{P}_{1\perp}| |\mathbf{P}_{2\perp}|}{m_1 m_2} \mathcal{C}[w_1 \hat{t}_1^h H_1^\perp] \quad (11)$$

$$\mathcal{F}_{UU}^{\cos(2\phi_1)} = \frac{\mathbf{P}_{1\perp}^2}{m_1 m_N} \mathcal{C}[w_2 \hat{t}_1^\perp H_1^\perp] \quad (12)$$

$$\begin{aligned} \mathcal{F}_{UU}^{\cos(2\phi_2)} &= \frac{\mathbf{P}_{2\perp}^2}{m_1 m_2} \mathcal{C}[w_3 \hat{t}_1^h H_1^\perp] \\ &+ \frac{\mathbf{P}_{2\perp}^2}{m_1 m_N} \mathcal{C}[w_4 \hat{t}_1^\perp H_1^\perp] \end{aligned} \quad (13)$$

$$\mathcal{F}_{LU}^{\sin(\phi_1 - \phi_2)} = \frac{|\mathbf{P}_{1\perp}| |\mathbf{P}_{2\perp}|}{m_N m_2} \mathcal{C}[w_5 \hat{t}_1^h D_1], \quad (14)$$

with the following notation for the transverse momentum convolution

$$\begin{aligned} \mathcal{C}[f(\mathbf{k}_\perp, \mathbf{k}'_\perp, \dots)] &\equiv \sum_a e_a^2 x_B \int d^2\mathbf{k}_\perp \int d^2\mathbf{k}'_\perp \\ &\times \delta^2(\mathbf{k}_\perp - \mathbf{k}'_\perp - \mathbf{P}_{1\perp}/z_1) f(\mathbf{k}_\perp, \mathbf{k}'_\perp, \dots). \end{aligned} \quad (15)$$

In Eqs. (11-14) the  $w_i$ 's are scalar functions of the vectors  $\mathbf{k}_\perp$ ,  $\mathbf{k}'_\perp$ ,  $\mathbf{P}_{1\perp}$ ,  $\mathbf{P}_{2\perp}$ , whose explicit expressions will be given in [16] (in the sequel, we will write down explicitly the only coefficient we need here, *i.e.*  $w_5$ ).

If we integrate the cross section over  $\phi_2$ , keeping  $\Delta\phi \equiv \phi_1 - \phi_2$  fixed, the only two surviving structure functions in (9) are  $\mathcal{F}_{UU}$  and  $\mathcal{F}_{LU}^{\sin \Delta\phi}$  (the transverse spin of quarks in this case plays no role). The beam spin asymmetry  $\mathcal{A}_{LU}$ , defined as

$$\mathcal{A}_{LU}(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2, \Delta\phi) = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}}, \quad (16)$$

is given by

$$\begin{aligned} \mathcal{A}_{LU} &= - \frac{y \left(1 - \frac{y}{2}\right)}{\left(1 - y + \frac{y^2}{2}\right)} \frac{\mathcal{F}_{LU}^{\sin \Delta\phi}}{\mathcal{F}_{UU}} \sin \Delta\phi \\ &= - \frac{|\mathbf{P}_{1\perp}| |\mathbf{P}_{2\perp}|}{m_N m_2} \frac{y \left(1 - \frac{y}{2}\right)}{\left(1 - y + \frac{y^2}{2}\right)} \\ &\quad \times \frac{\mathcal{C}[w_5 \hat{t}_1^h D_1]}{\mathcal{C}[\hat{u}_1 D_1]} \sin \Delta\phi, \end{aligned} \quad (17)$$

with

$$w_5 = \frac{(\mathbf{k}_\perp \cdot \mathbf{P}_{2\perp})(\mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp}) - (\mathbf{k}_\perp \cdot \mathbf{P}_{1\perp})\mathbf{P}_{2\perp}^2}{(\mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp})^2 - \mathbf{P}_{1\perp}^2 \mathbf{P}_{2\perp}^2}. \quad (18)$$

Let us consider now the most general azimuthal dependence of Eq. (17), in order to extract a clear signature for the spin-beam asymmetry  $\mathcal{A}_{LU}$ . One should remember that all structure functions can carry a dependence on  $\mathbf{P}_{1\perp} \cdot \mathbf{P}_{2\perp} = |\mathbf{P}_{1\perp}| |\mathbf{P}_{2\perp}| \cos \Delta\phi$ , arising from a transverse momentum correlation of the type  $\mathbf{k}_\perp \cdot \mathbf{P}_{2\perp}$  in the fracture functions. It is reasonable to assume that these correlations are small, so that we can expand the fracture functions in powers of  $\mathbf{k}_\perp \cdot \mathbf{P}_{2\perp}$  keeping only the first few terms; for instance:

$$\begin{aligned} \hat{t}_1^h(x_B, \zeta_2, \mathbf{k}_\perp^2, \mathbf{P}_{2\perp}^2, \mathbf{k}_\perp \cdot \mathbf{P}_{2\perp}) \\ \simeq a(x_B, \zeta_2, \mathbf{k}_\perp^2, \mathbf{P}_{2\perp}^2) \\ + b(x_B, \zeta_2, \mathbf{k}_\perp^2, \mathbf{P}_{2\perp}^2) \mathbf{k}_\perp \cdot \mathbf{P}_{2\perp}. \end{aligned} \quad (19)$$

The term linear in  $\mathbf{k}_\perp \cdot \mathbf{P}_{2\perp}$  yields a  $\cos \Delta\phi$  term in the structure functions, which, combined with the  $\sin \Delta\phi$  term already explicitly appearing in Eq. (17), results in the following angular dependence for the beam-spin asymmetry:

$$\begin{aligned} \mathcal{A}_{LU}(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2, \Delta\phi) \\ = A(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2) \sin \Delta\phi \\ + B(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2) \sin(2\Delta\phi). \end{aligned} \quad (20)$$

These two azimuthal modulations are typical of  $\mathcal{A}_{LU}$  and would be a clear signature of its presence; they both originate from a correlation between the quark transverse momentum  $\mathbf{k}_\perp$  and the hadron transverse momentum  $\mathbf{P}_{2\perp}$ , resulting in a long range correlation between  $\mathbf{P}_{1\perp}$ , the momentum of the hadron in the CFR, and  $\mathbf{P}_{2\perp}$ , the momentum of the hadron in the TFR, which yields a specific and unambiguous dependence on  $\phi_1 - \phi_2$ .

We have shown that a new beam-spin asymmetry appears, at leading twist and low transverse momenta, in the deep inelastic inclusive lepto-production of two hadrons, one in the target fragmentation region and one in the current fragmentation region. Such a single spin asymmetry, if sizable, cannot originate from pQCD effects and must be related to non perturbative properties of partonic distributions. When interpreted in the

QCD parton model and in a non collinear factorization scheme (TMD factorization) based on fracture and fragmentation functions, the asymmetry has a clear partonic origin in a correlation between the quark intrinsic motion and the transverse motion of the hadron produced in the TFR. The TMD factorization has been widely used in the CFR to obtain information on TMDs and the 3-dimensional momentum structure of the nucleon, and this is known to hold in the TFR as well [17–19]

The new beam-spin asymmetry introduced here has a definite and clear signature which can be experimentally tested, both in running experiments (JLab) and future ones (upgraded JLab and future electron-ion or electron-nucleon colliders, EIC/ENC). If experimentally observed, it would confirm the validity of the TMD factorization in high energy lepto-production for TFR events, thus opening new ways of exploring the nucleon internal structure.

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